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Aircraft Concept Optimization Using Parametric Multiobjective Figures of Merit

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Introduction

THE difference between aircraft optimized for different figures of merit is of interest to aircraft designers, e.g., the contrast between a minimum takeoff gross weight design and a minimum fuel weight design. Results obtained using different single-objective figures of merit were compared by Jensen et al.[1] and Johnson.[2] The results from single figure of merit studies are useful, but do not give a clear picture of how a design changes as it moves from one figure of merit to another. By examining the evolution from one design to another, a deeper understanding of the role of the figure of merit can be gained. In this Note we extend our previous work[3] to handle several figures of merit through the use of a multiobjective figure of merit approach, and show how this approach can be used to gain additional insight into the relative importance of various figures of merit. Using this method, the evolution of an optimum design from one figure of merit to another is demonstrated. Specifically, we have used combinations of minimum takeoff gross weight, structural weight, fuel weight, maximum cruise performance, and productivity parameters as figures of merit.[4] We use the global sensitivity equation (GSE) approach, and computational speed is facilitated by the use of simple algebraic representations of the system technologies.

Global Sensitivity Approach and Algebraic Technology Representations

Sets of optimum designs for various figures of merit can be obtained quickly using the multidisciplinary methods developed by Sobieski,[5] and by taking advantage of the low computational time afforded by the use of algebraic expressions for various technology models. The approach is structured so that the algebraic models can be replaced by improved analysis as desired. An alternate, much more efficient approach to incorporation of more advanced analysis methods is possible using the variable-complexity approach.[6]

The GSE method is used to determine the interactions between the system technologies. The gradients of the figure of merit and constraints are available from the GSE analysis. The solution to the numerical optimization problem is obtained using a nonlinear quadratic programming algorithm[7] with the GSE gradients supplied for the search direction computation. Complete details of the method are given in Ref. 3.

Parametric Multiobjective Function Formulation

As described previously, we define a range of figures of merit for use in multiple optimization cases to illustrate the evolution of a design from one figure of merit to another. This is done by defining a multiobjective figure of merit F in the following manner:

$$F = K_0 F_{\mathrm{obj1}} + (1 - K_0) F_{\mathrm{obj2}} \tag{1}$$

where K_0 is a blending parameter that varies from 0 to 1. When $K_0 = 1$, the figure of merit is the first figure of merit F_{obj1}, and when $K_0 = 0$, the figure of merit is entirely F_{obj2}. When K_0 is between 1 and 0, the overall figure of merit is a combination of the two.

Illustrative Example

Baseline Mission and Design Variable Set

Our parametric multiobjective function approach is illustrated using a short takeoff, medium-range cargo transport. Table 1 gives the specified mission statement and constraints along with a definition of the propulsion system. In this example, the range and the engine thrust were fixed. The problem was studied previously in Ref. 3 for a single objective function.

The design variables were chosen to include both aircraft geometry and flight performance values. Seven design variables were chosen for optimization:

$$X = (\mathrm{AR}, S_w, h, M, \Lambda, t/c, \lambda)^T \tag{2}$$

where AR is the aspect ratio, S_w is the wing area, h is the cruise altitude, M is the Mach number, Λ is the wing sweep angle, t/c is the wing thickness to chord ratio, and λ is the wing taper ratio. Variations in the optimum values of the design variables along with the figure of merit values will be examined as the parameter K_0 is varied. The results provide insight into how the aircraft geometry is affected by changing the figure of merit, and what combinations of these figures of merit result in the most robust aircraft design for various mission profiles.

Solutions for Minimizing Fuel Weight and Wing Weight

As an example, we select the cruise fuel weight and the wing structural weight as the two figures of merit to use in our multiobjective figure of merit. Other examples are given in Ref. 4. The formulation for the multiobjective function for this case is given by

$$F = K_0 W_{\mathrm{wing}} + (1 - K_0) W_{\mathrm{fuel}} \tag{3}$$

where W_{wing} is the wing weight, and W_{fuel} is the fuel weight. A takeoff constraint of 5000 ft is imposed along with a section

Table 1 Mission requirements/constraints

Cargo weight, 150,000 lb
Range, 3,000 nm
Takeoff distance, 5,000 ft
Landing distance, 4,000 ft
Maximum C_L, 2.3
Maximum section $C_{l_{\mathrm{cruise}}}$, 1.0
Propulsion, 4 CF6 class turbofans

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lift coefficient limit. By prescribing the figure of merit in the fashion shown previously, the design can be selectively optimized to minimize the wing weight when K_0 is 1, the fuel weight when K_0 is 0, or any combination of the two when K_0 is between 1 and 0. The resulting solutions reflect the shift from a fuel-minimized, cruise-efficient design, to a structurally efficient design as K_0 is increased from 0 to 1. This tradeoff illustrates the importance of proper objective function selection and the relative influence of each component when the total aircraft weight is used as a figure of merit.

Figures 1–3 show the results of using Eq. (3) as the objective function in the optimization. Figure 1 gives fuel, wing, and takeoff gross weight results for K_0 going from 0 to 1.

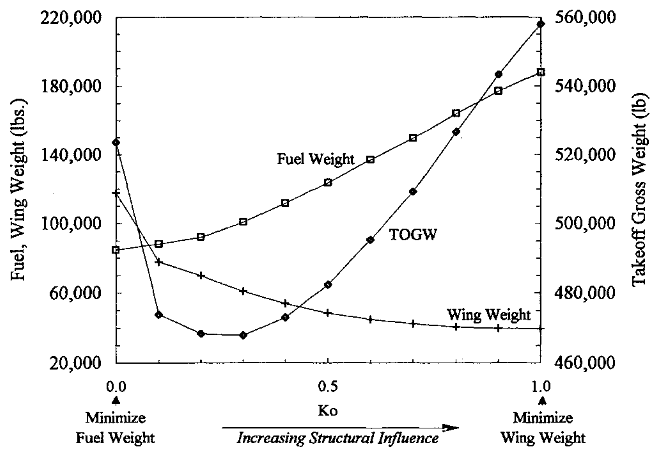

Fig. 1 Component weights.

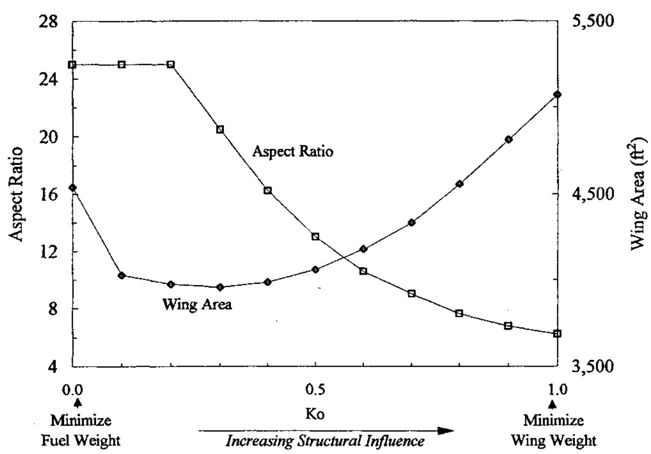

Fig. 2 Wing area and aspect ratio.

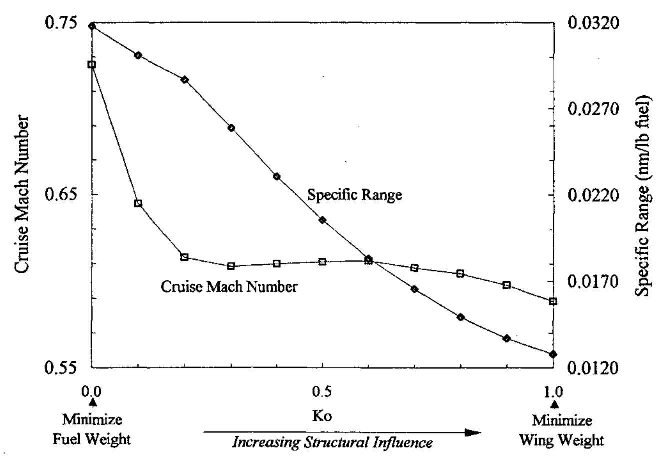

Fig. 3 Mach number and specific range.

There is a large penalty in overall gross weight when either the fuel or wing weight is minimized alone. The relative contributions from the wing weight and the fuel weight to the total takeoff weight formulation are also well illustrated. For $K_0 = 0.2$, W_{to} is a minimum. This shows that if W_{to} were used as the objective function, the result could be interpreted as 20% influence of wing structural weight and 80% fuel weight. Moreover, the designer can get some insight into the sensitivity of the takeoff gross weight, or any other design characteristic, to a change of the weighting in favor of either structural weight or fuel weight.

Figure 2 presents the corresponding optimum aspect ratio and wing area results. For the minimum fuel weight solution the aspect ratio is at the upper limit of 25, and the wing area is slightly less than 4600 ft^2. As the influence of the wing structural weight is increased in the objective function, the design shifts to a low aspect ratio design. The wing area decreases as K_0 moves from 0.0 to 0.2, where takeoff gross weight is a minimum, and then increases to a maximum of just over 5000 ft^2 at $K_0 = 1$. The decreasing aspect ratio is the result of the penalty for large aspect ratios in the wing weight equation.

Figure 3 presents the resulting variation in specific range and Mach number. As expected, the specific range is at its largest value when fuel weight is minimized. As the influence of the wing structural weight increases the specific range decreases continually. The optimum cruise Mach number changes from 0.72 ($K_0 = 0$) to just under 0.60 ($K_0 = 1$), with most of the decrease occurring immediately as K_0 increases above 0. The Mach number decrease occurs mainly because of a reduction in wing sweep (not shown here), which reduces the wing weight significantly. At reduced wing sweep the cruise Mach number decreases to avoid transonic drag rise.

Conclusions

A new and insightful method has been presented that addresses the concerns of the configuration designer faced with considering design for multiple figures of merit. The behavior of the optimal solution over a range of multiobjective functions has been presented and compared to solutions with single objective functions. The influences from several single figures of merit as observed within a multiobjective optimization problem illustrate the wide range of configurations that can arise from such problem formulations. Finally, the advantage of the multidisciplinary approach coupled with the speed of algebraic technology representations was exemplified in the large number of optimal solutions computed.

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